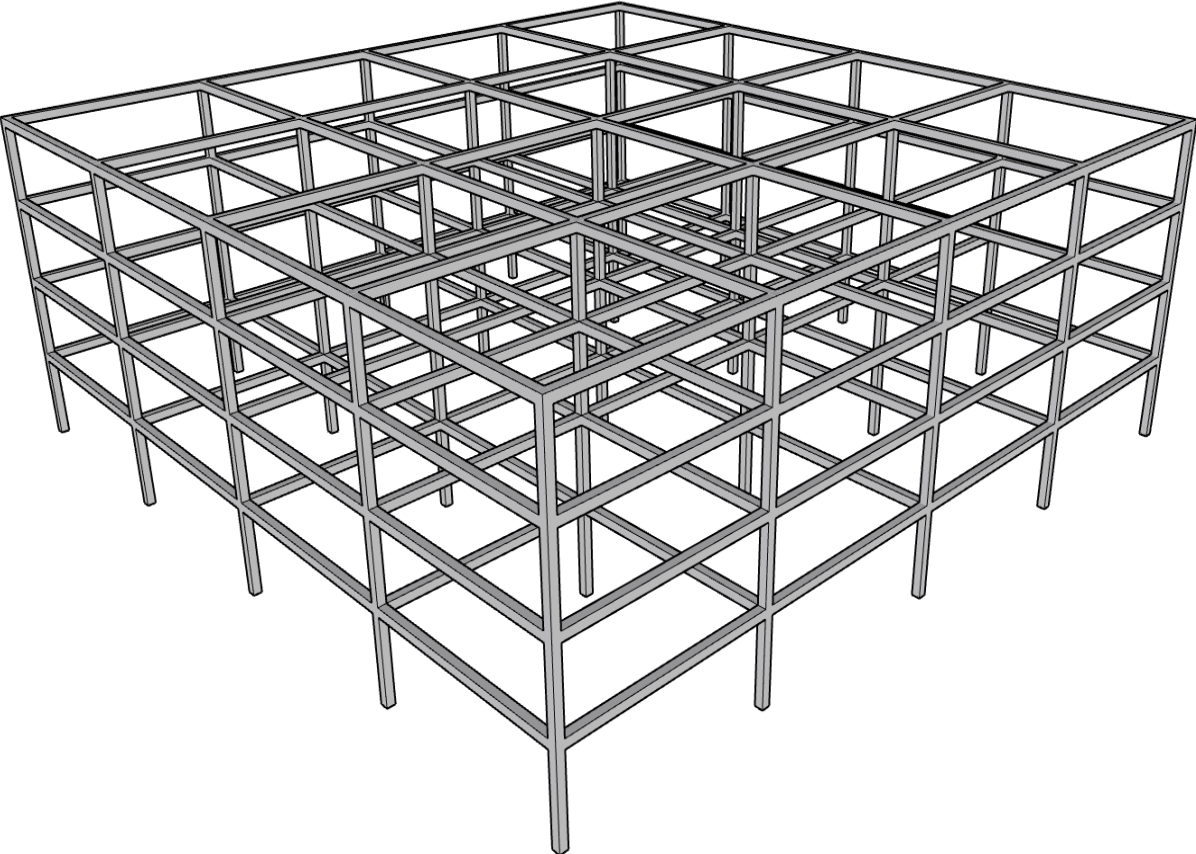
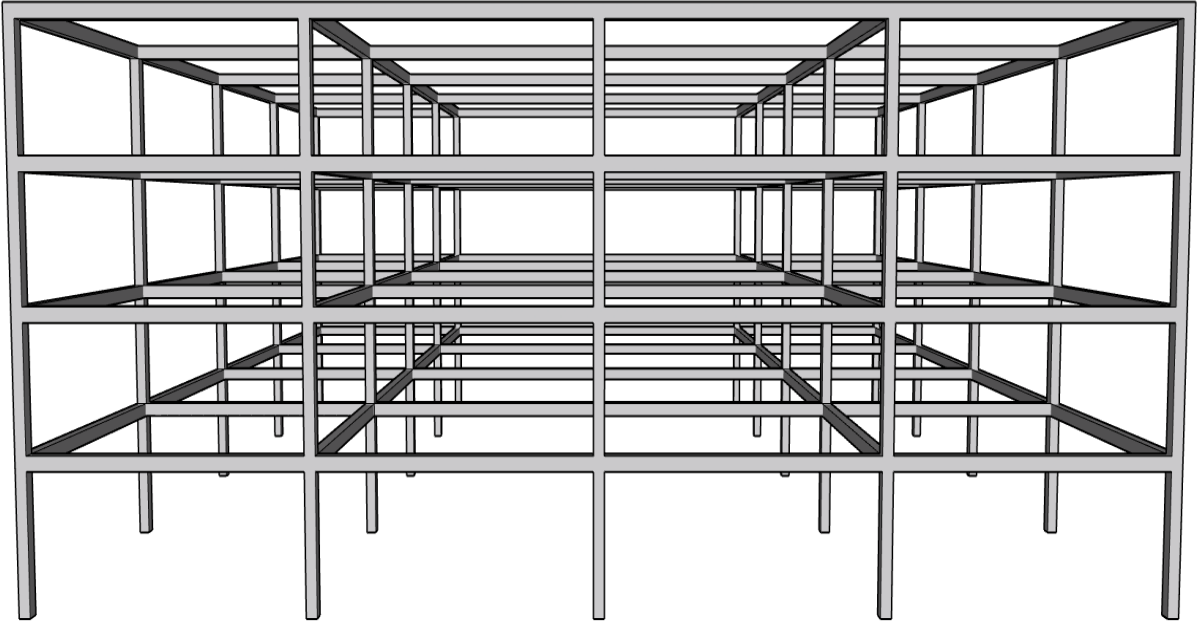
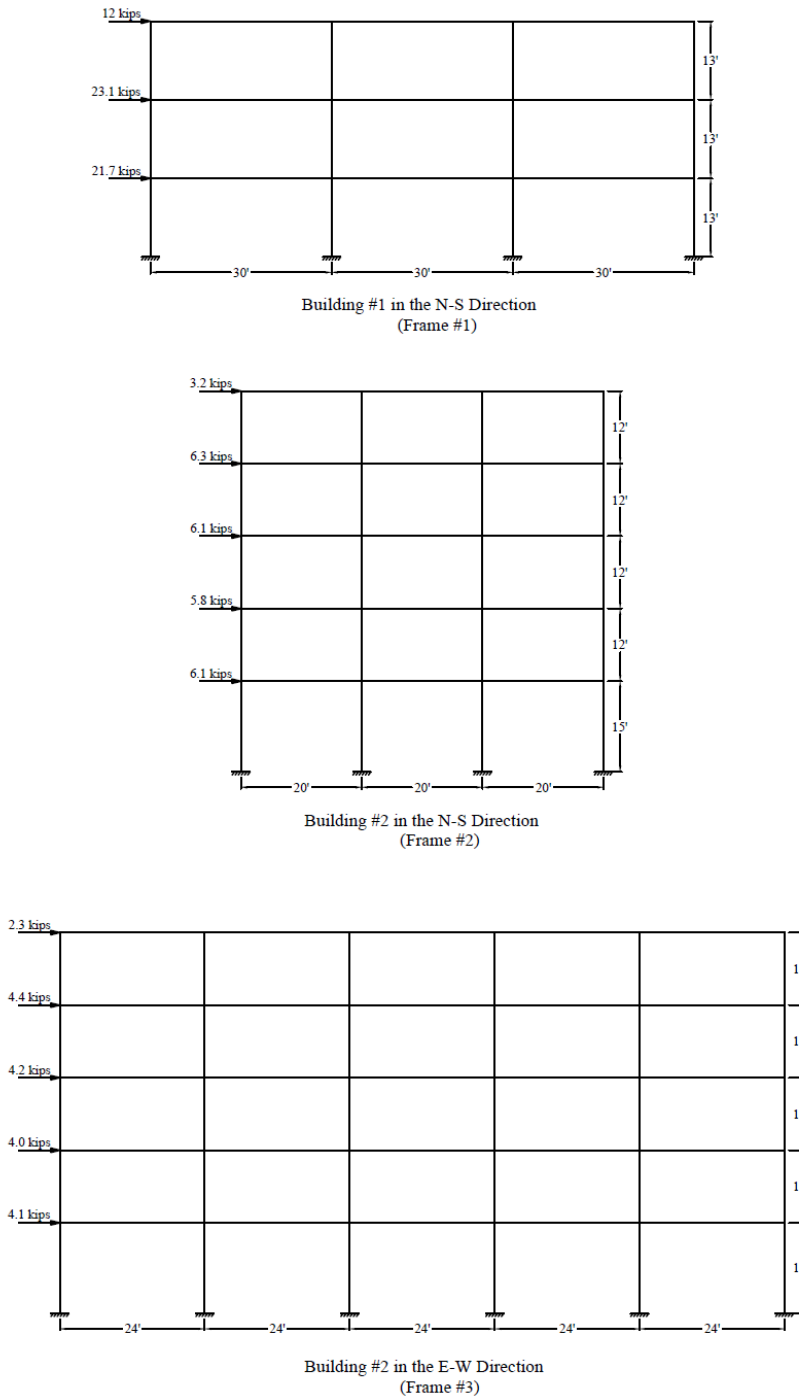


**Structural Analysis of Reinforced Concrete Frames**



### Structural Analysis of Reinforced Concrete Frames

The moments, shears, and axial forces using the Portal Method are determined for the following frames resulting from wind loads acting in the directions shown in the figures. The wind loads are determined using ASCE 7-10 provisions. The results of hand calculations are then compared with numerical analysis results obtained from the [spFrame](#) engineering software program.



**Figure 1 – Reinforced Concrete Continuous Beams at intermediate building floor**

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## Contents

|   |    |
|---|----|
| 1. Columns Shear Forces.....                              | 3  |
| 2. Columns Axial Loads.....                               | 3  |
| 3. Columns Moments .....                                  | 3  |
| 4. Beams Shear Forces and Moments .....                   | 3  |
| 5. Static Equilibrium Check .....                         | 5  |
| 6. Structural Analysis of Frames – spFrame Software ..... | 7  |
| 7. Results Comparison and Conclusions .....               | 13 |

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## Reference

Simplified Design of Reinforced Concrete Buildings, Fourth Edition, 2011 Portland Cement Association  
[spFrame](#) Engineering Software Program Manual v1.50a, STRUCTUREPOINT, 2012

## Analysis Data

The applied lateral wind loads are calculated based on the provisions of American Society of Civil Engineers Minimum Design Loads for Buildings and Other Structures (ASCE/SEI 7).

All loads and dimensions are shown in Figure 1.

Portal Method is used for the frame analysis.

**Solution**

The Portal Method considers a two-dimensional frame consisting of a line of columns and their connecting horizontal members (slabs-beams), with each frame extending the full height of the building. The frame is considered to be a series of portal units. Each portal unit consists of two story-high columns with connecting slab-beams. Points of contraflexure are assumed at mid-length of beams and mid-height of columns. The following figure illustrates the portal unit concept applied to the top story of a building frame, with each portal unit shown separated (but acting together).

The following analysis calculations are for Frame #1. The same procedure is used to get moments, shears, and axial forces for Frames #2 and #3.

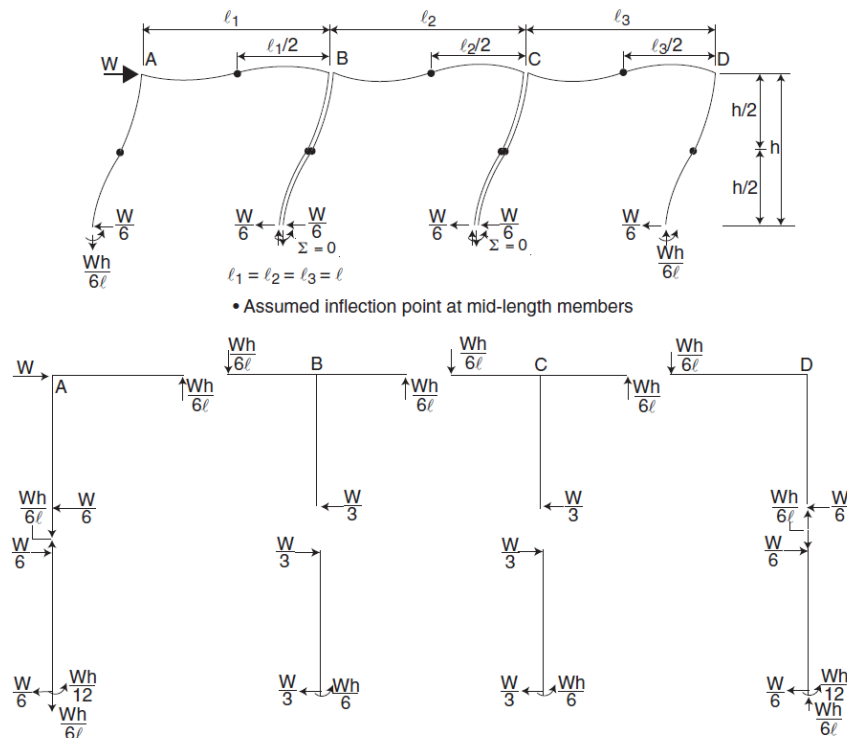


Figure 2 – Portal Method

## 1. Columns Shear Forces

For the end columns

$$3^{\text{rd}} \text{ story: } V = \frac{12}{6} = 2.00 \text{ kips}$$

$$2^{\text{nd}} \text{ story: } V = \frac{12 + 23.1}{6} = 5.85 \text{ kips}$$

$$1^{\text{st}} \text{ story: } V = \frac{12 + 23.1 + 21.7}{6} = 9.50 \text{ kips}$$

For the interior columns

$$3^{\text{rd}} \text{ story: } V = \frac{12}{3} = 4.00 \text{ kips}$$

$$2^{\text{nd}} \text{ story: } V = \frac{12 + 23.1}{3} = 11.70 \text{ kips}$$

$$1^{\text{st}} \text{ story: } V = \frac{12 + 23.1 + 21.7}{3} = 19.00 \text{ kips}$$

## 2. Columns Axial Loads

For end columns, the axial loads can be obtained by summing moments about the column inflection points at each level. For example, for the 2<sup>nd</sup> story columns:

$$\sum M = 0$$

$$12 \times \left( 13 + \frac{13}{2} \right) + 23.1 \times \frac{13}{2} - P \times (30 + 30 + 30) = 0$$

$$P = 4.27 \text{ kips}$$

## 3. Columns Moments

Moments can be obtained by multiplying the column shear force by one-half of the column length. For example, for an exterior column in the 2<sup>nd</sup> story:

$$M = 5.85 \times \frac{13}{2} = 38.03 \text{ kips-ft}$$

## 4. Beams Shear Forces and Moments

Shear and moments in the beams can be obtained by satisfying equilibrium at each joint. The following figure shows the free-body diagrams for the 2<sup>nd</sup> story

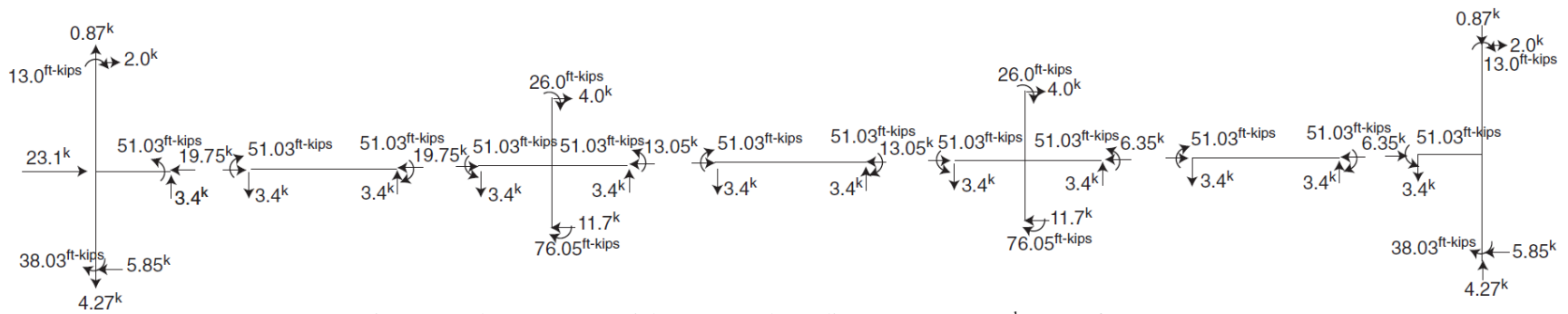


Figure 3 – Shear Forces, Axial Forces, and Bending Moments at 2<sup>nd</sup> Story of Frame #1

### 5. Static Equilibrium Check

As a final check, the summation of moments about the base of the frame must be equal to zero:

$$\sum M = 0$$

$$12 \times (13 + 13 + 13) + 23.1 \times (13 + 13) + 21.7 \times 13 - 10.91 \times (30 + 30 + 30)$$

$$-P \times (30 + 30 + 30) - 2 \times (61.53 + 123.07) = 0 \rightarrow o.k.$$

The same procedure is performed for Frames #2 and #3. The following figures show a summary of wind load analysis for the three frames.

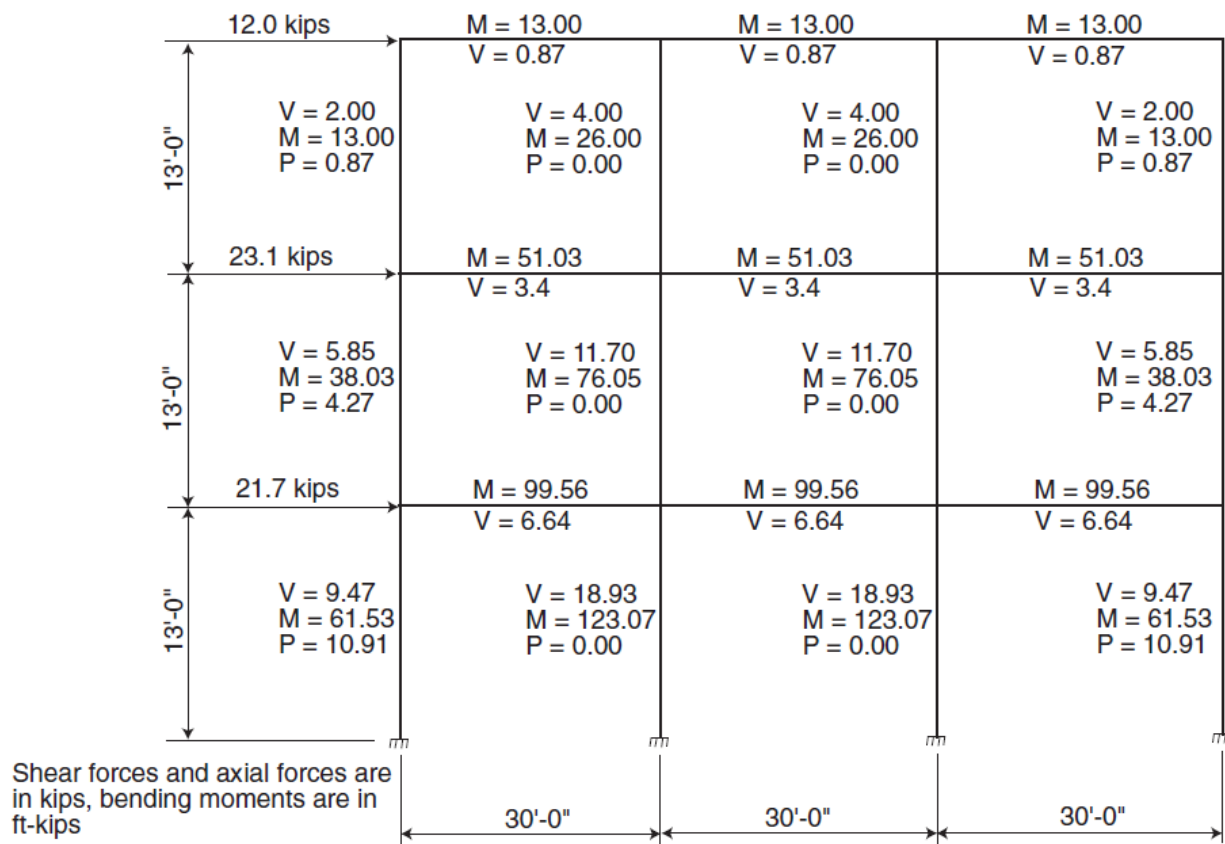


Figure 4 – Shear, Moments, and Axial Forces for Frame #1 Using the Portal Method



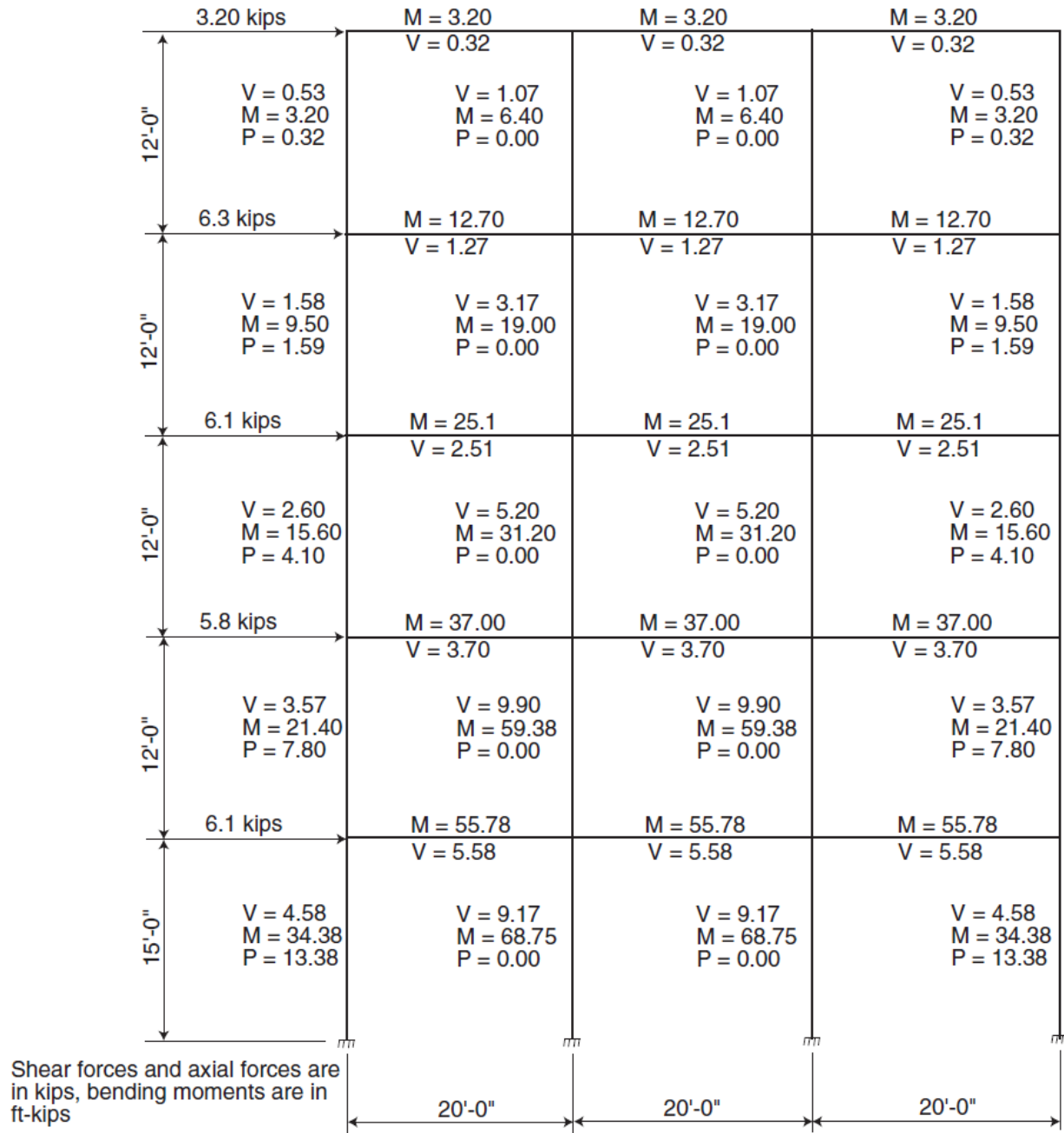


Figure 5 – Shear, Moments, and Axial Forces for Frame #2 Using the Portal Method

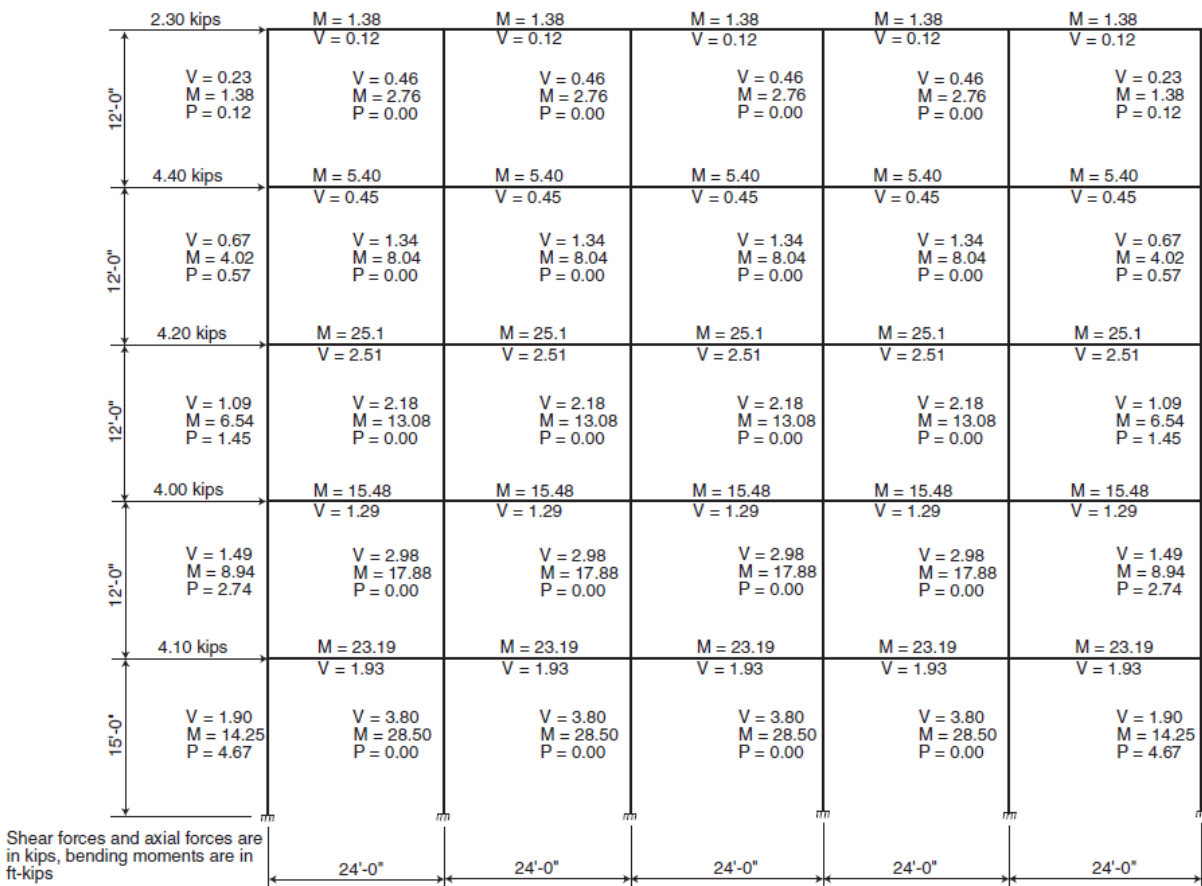


Figure 6 – Shear, Moments, and Axial Forces for Frame #3 Using the Portal Method

## 6. Structural Analysis of Frames – spFrame Software

spFrame is used to analyze two and three-dimensional frame or truss structures subjected to static loads. The structure to be analyzed must be reduced to a mathematical model represented by straight lines. These lines are referred to as members. Member ends are referred to as joints or nodes.

spFrame offers flexibility through its impressive array of features which include

- Practically unlimited number of joints and members.
- Second or (P-Δ) effects and tension-only members.
- Proper shear wall modeling.
- Hinged or partially-fixed element ends.
- Shear deformations.
- Translational and rotational elastic supports.
- Inclined supports.
- Rigid-end offsets.

- Rigid diaphragms.
- Non-prismatic members.
- Loads may be applied to
  - Members—as point (force or moment), uniform, and triangular loads.
  - Members—as caused by temperature effects.
  - Joints—as forces, moments, and known displacement

Joints and members are assigned physical properties to simulate the real structure. A joint can be assigned properties such as restraining its rotation about a particular direction, restraining its movement in a particular direction or to a certain path, controlling the magnitude of its movement, or rotation, if a force or moment occurs at that joint. A member is assigned location, orientation, material properties and section properties. Further refinement of the mathematical model can be achieved by the simulation of non-prismatic members, applying member end partial releases, accounting for rigid end offsets, shear deformations, and the stiff diaphragm action. Loads are categorized into load cases that can be factored and combined under load combinations. The modeled structure is then analyzed under these load combinations and the resulting forces, moments, displacements, and rotations are presented for each load combination.

For illustration and comparison purposes, the following figures provide a sample of the input modules and results obtained from an spFrame model created for the three frames used in this example.

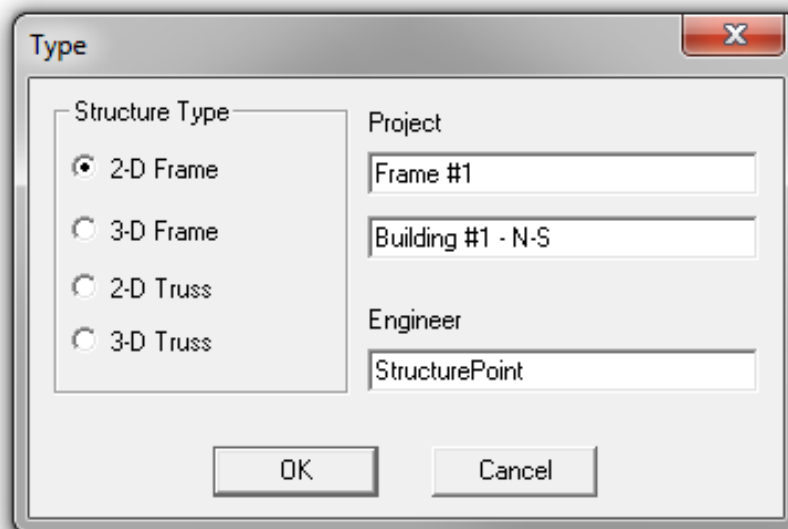


Figure 7 – Element Type (spFrame)

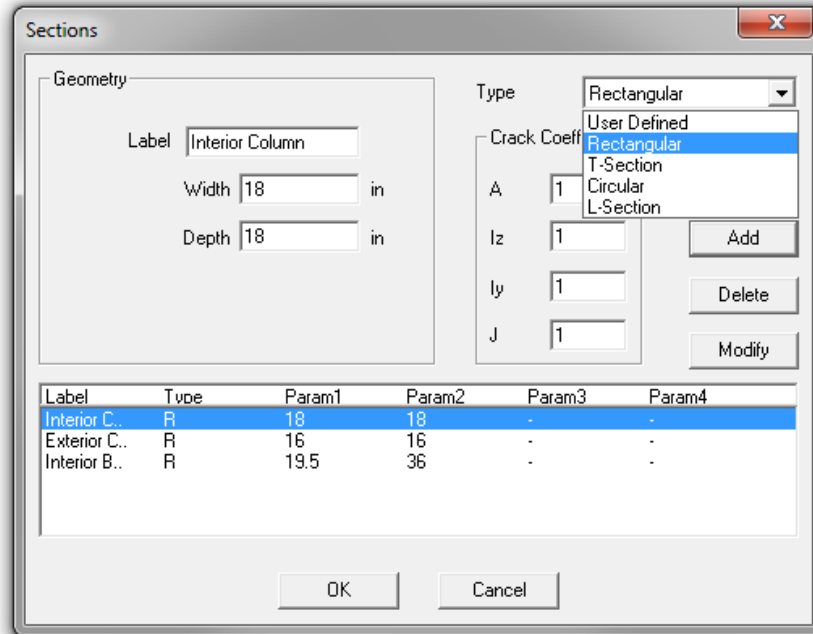


Figure 8 – Defining Sections (spFrame)

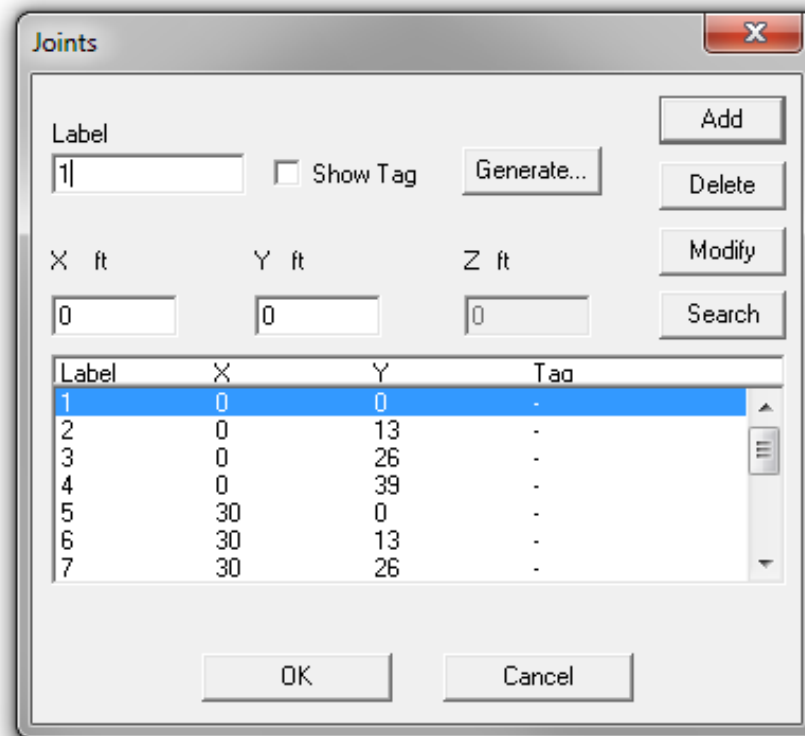


Figure 9 – Defining Joints (spFrame)

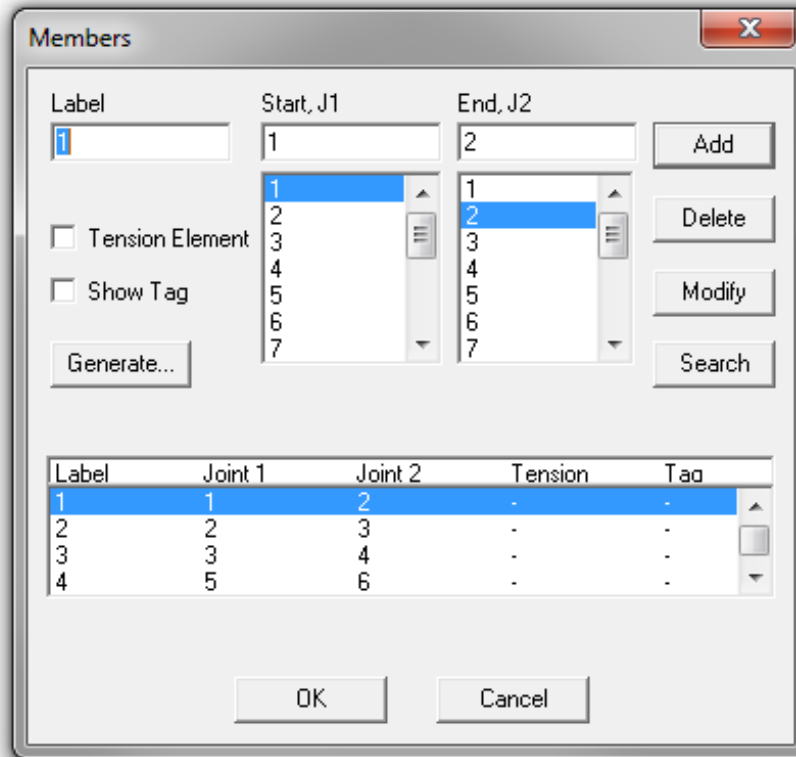


Figure 10 – Defining Members (spFrame)

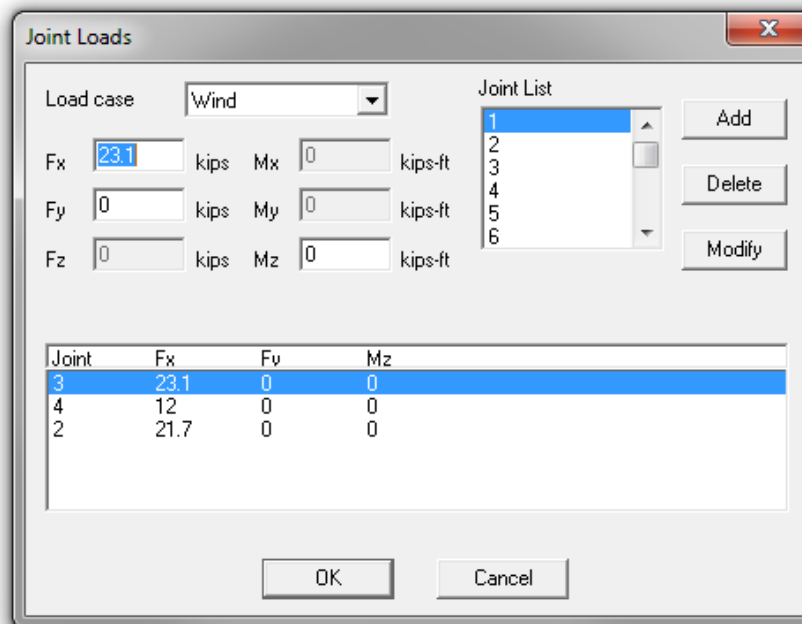


Figure 11 – Defining Joint Loads (spFrame)

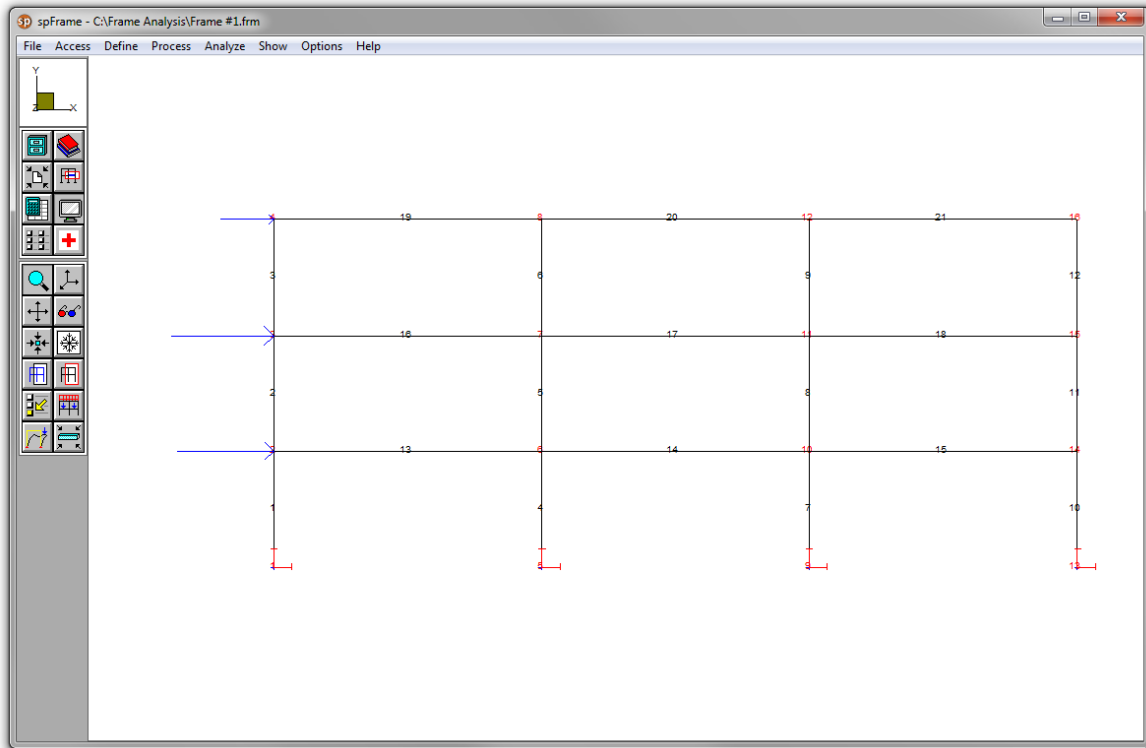


Figure 12 – Joints, Members, Fixities, and Load Diagrams (spFrame)

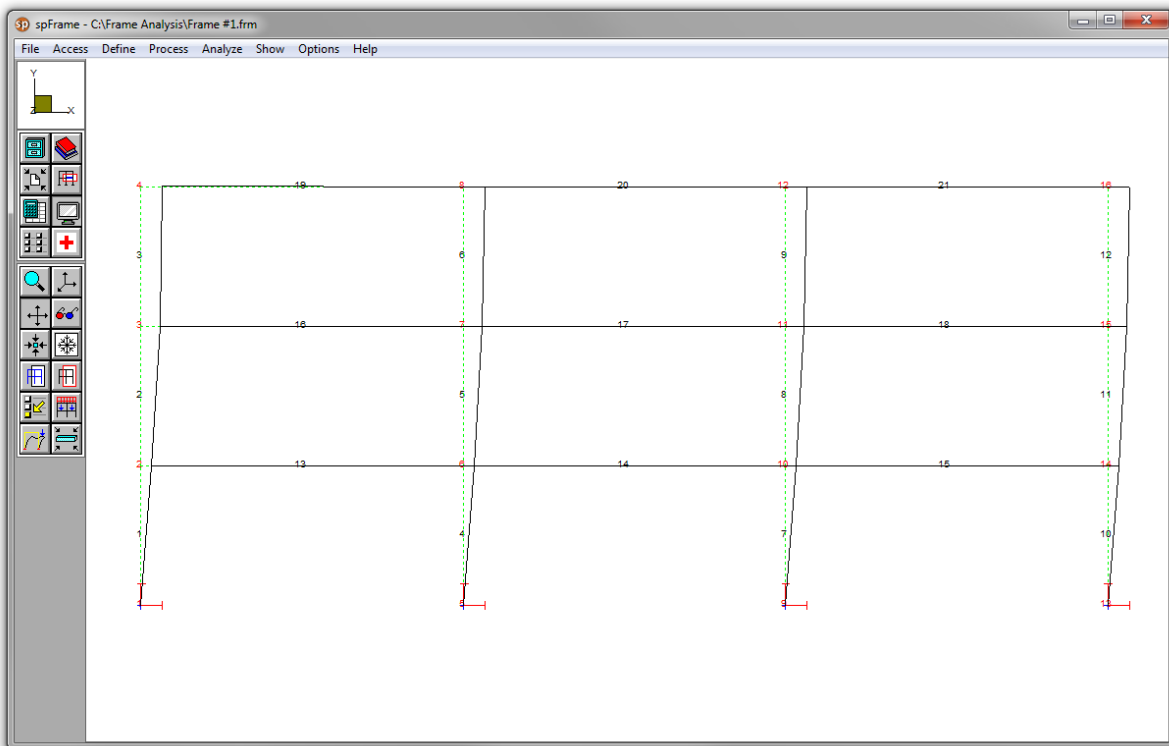


Figure 13 – Frame Deformed Shape (spFrame)

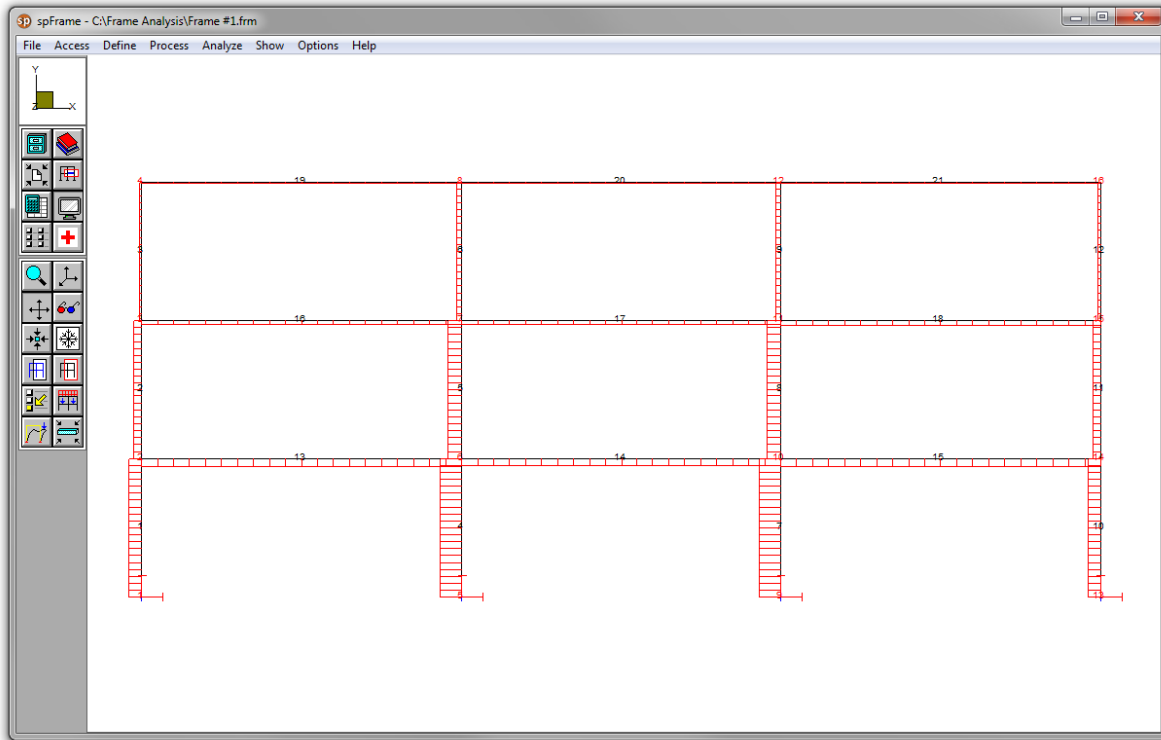


Figure 14 – Shear Diagrams (spFrame)

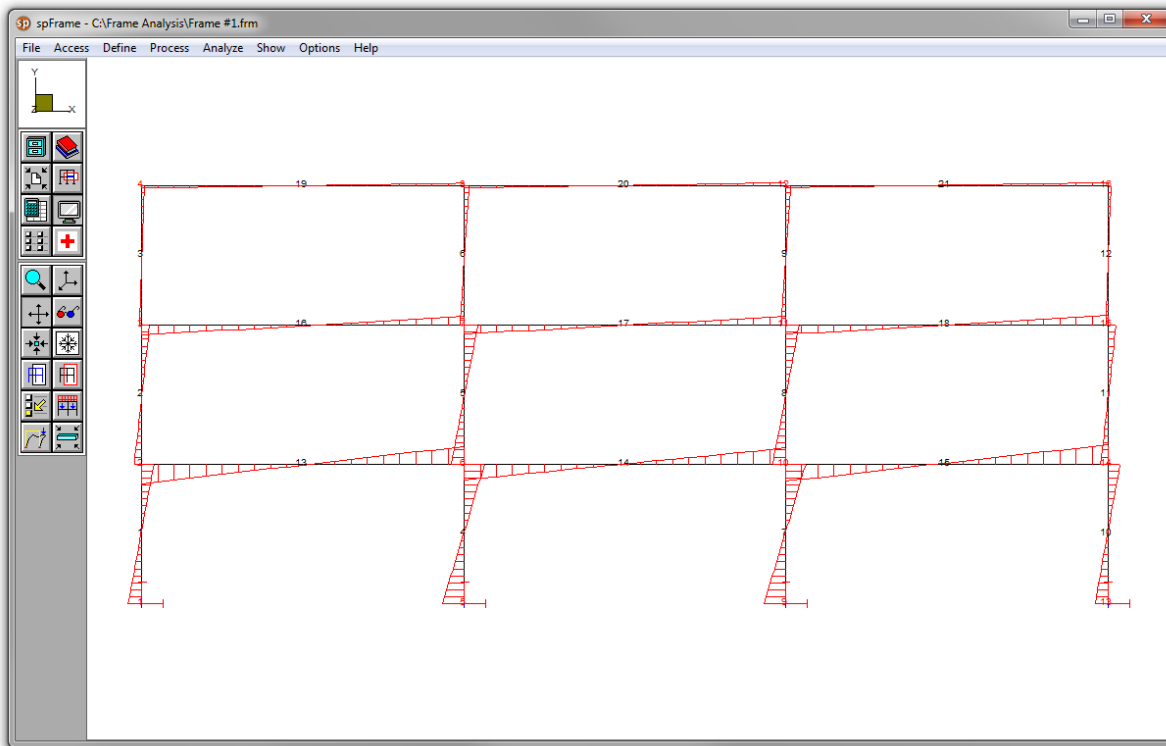


Figure 15 – Moment Diagrams (spFrame)

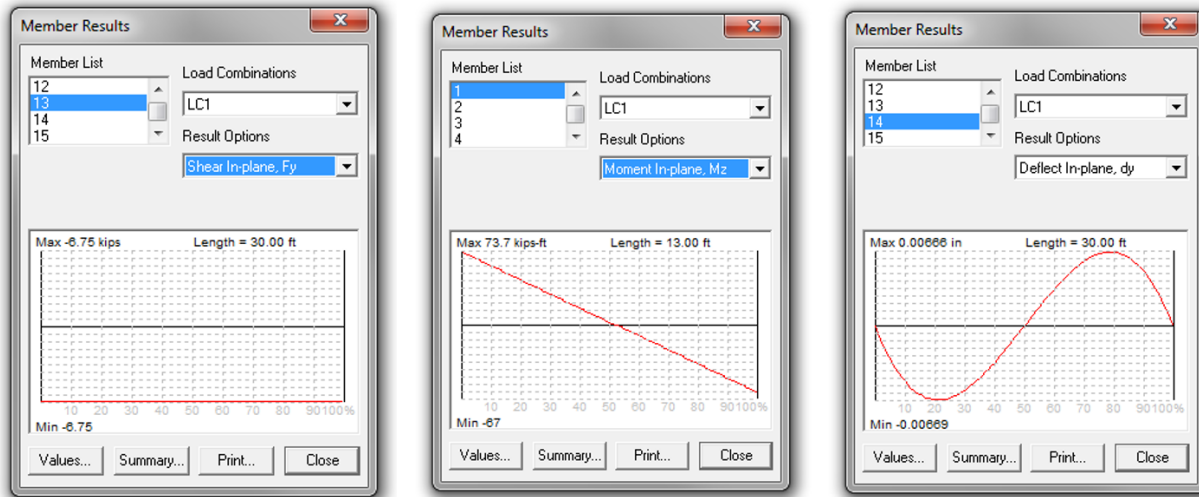


Figure 16 – Member Results (spFrame)

## 7. Results Comparison and Conclusions

The following figures and tables show the comparison between hand results and [spFrame](#) model results.

For Frame #1:

|   |    |   |    |   |    |
|---|----|---|----|---|----|
|   | 19 |   | 20 |   | 21 |
| 3 |    | 6 |    | 9 | 12 |
|   | 16 |   | 17 |   | 18 |
| 2 |    | 5 |    | 8 | 11 |
|   | 13 |   | 14 |   | 15 |
| 1 |    | 4 |    | 7 | 10 |

Figure 17 – Element Numbering for Frame #1 (spFrame)



Table 1 - Comparison of Axial, Shear, and Moment Forces (Frame #1)

| Member | P, kip        |                         | V, kip        |                         | M, kip-ft     |                         |        |        |
|--------|---------------|-------------------------|---------------|-------------------------|---------------|-------------------------|--------|--------|
|        | Portal Method | <a href="#">spFrame</a> | Portal Method | <a href="#">spFrame</a> | Portal Method | <a href="#">spFrame</a> |        |        |
|        |               | M <sub>1</sub>          |               | M <sub>2</sub>          |               | Average                 |        |        |
| 1      | 10.91         | 11.20                   | 9.47          | 10.80                   | 61.53         | 73.70                   | 67.00  | 70.35  |
| 2      | 4.27          | 4.40                    | 5.85          | 6.37                    | 38.03         | 39.80                   | 43.00  | 41.40  |
| 3      | 0.87          | 0.90                    | 2.00          | 2.05                    | 13.00         | 12.10                   | 14.50  | 13.30  |
| 4      | 0.00          | 1.21                    | 18.93         | 17.80                   | 123.07        | 119.00                  | 112.00 | 115.50 |
| 5      | 0.00          | 0.25                    | 11.70         | 11.20                   | 76.05         | 71.00                   | 74.40  | 72.70  |
| 6      | 0.00          | 0.04                    | 4.00          | 3.86                    | 26.00         | 23.50                   | 26.60  | 25.05  |
| 7      | 0.00          | 1.20                    | 18.93         | 17.60                   | 123.07        | 118.00                  | 111.00 | 114.50 |
| 8      | 0.00          | 0.33                    | 11.70         | 11.20                   | 76.05         | 70.90                   | 74.40  | 72.65  |
| 9      | 0.00          | 0.00                    | 4.00          | 3.93                    | 26.00         | 24.00                   | 27.00  | 25.50  |
| 10     | 10.90         | 11.10                   | 9.47          | 10.60                   | 61.53         | 72.00                   | 65.40  | 68.70  |
| 11     | 4.27          | 4.47                    | 5.85          | 6.37                    | 38.03         | 39.90                   | 42.90  | 41.40  |
| 12     | 0.87          | 0.94                    | 2.00          | 2.17                    | 13.00         | 12.90                   | 15.30  | 14.10  |
| 13     |               |                         | 6.64          | 6.75                    | 99.56         | 95.70                   | 107.00 | 101.35 |
| 14     |               |                         | 6.64          | 5.80                    | 99.56         | 86.90                   | 87.00  | 86.95  |
| 15     |               |                         | 6.64          | 6.67                    | 99.56         | 105.00                  | 94.80  | 99.90  |
| 16     |               |                         | 3.40          | 3.50                    | 51.03         | 49.90                   | 55.10  | 52.50  |
| 17     |               |                         | 3.40          | 3.21                    | 51.03         | 48.10                   | 48.10  | 48.10  |
| 18     |               |                         | 3.40          | 3.53                    | 51.03         | 55.70                   | 50.30  | 53.00  |
| 19     |               |                         | 0.87          | 0.90                    | 13.00         | 12.50                   | 14.50  | 13.50  |
| 20     |               |                         | 0.87          | 0.94                    | 13.00         | 14.10                   | 14.10  | 14.10  |
| 21     |               |                         | 0.87          | 0.94                    | 13.00         | 15.30                   | 12.90  | 14.10  |

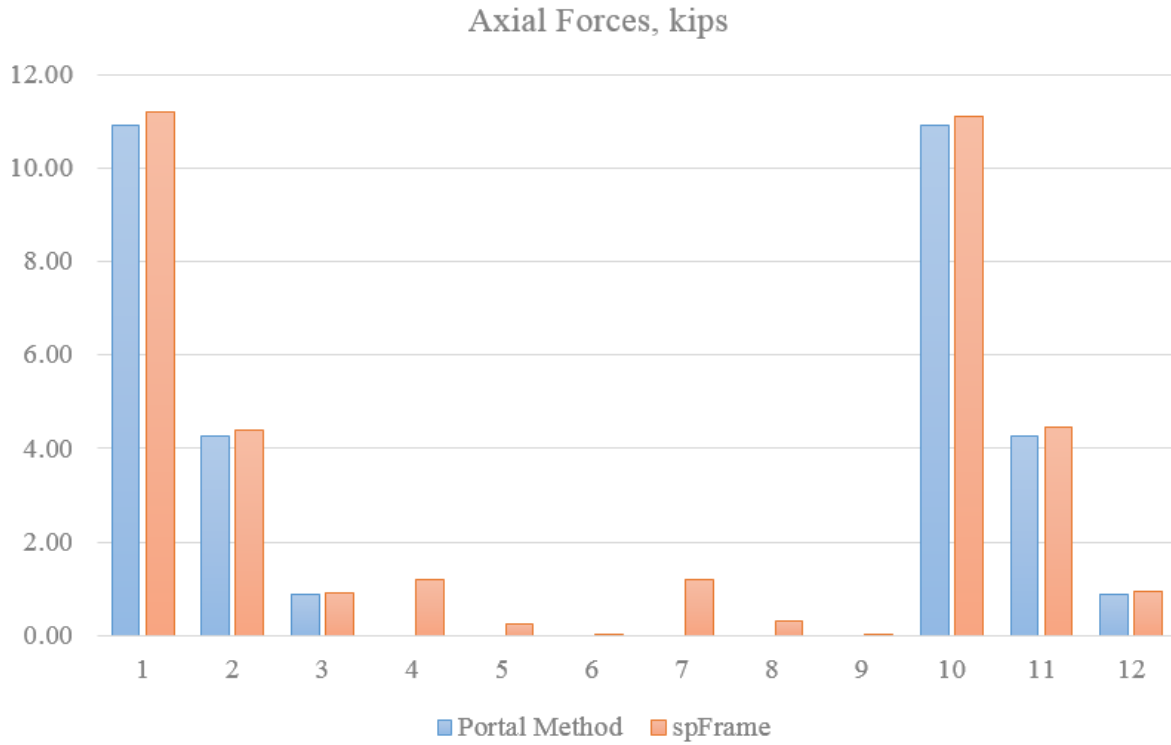


Figure 18 – Axial Forces for Frame #1 ([spFrame](#))

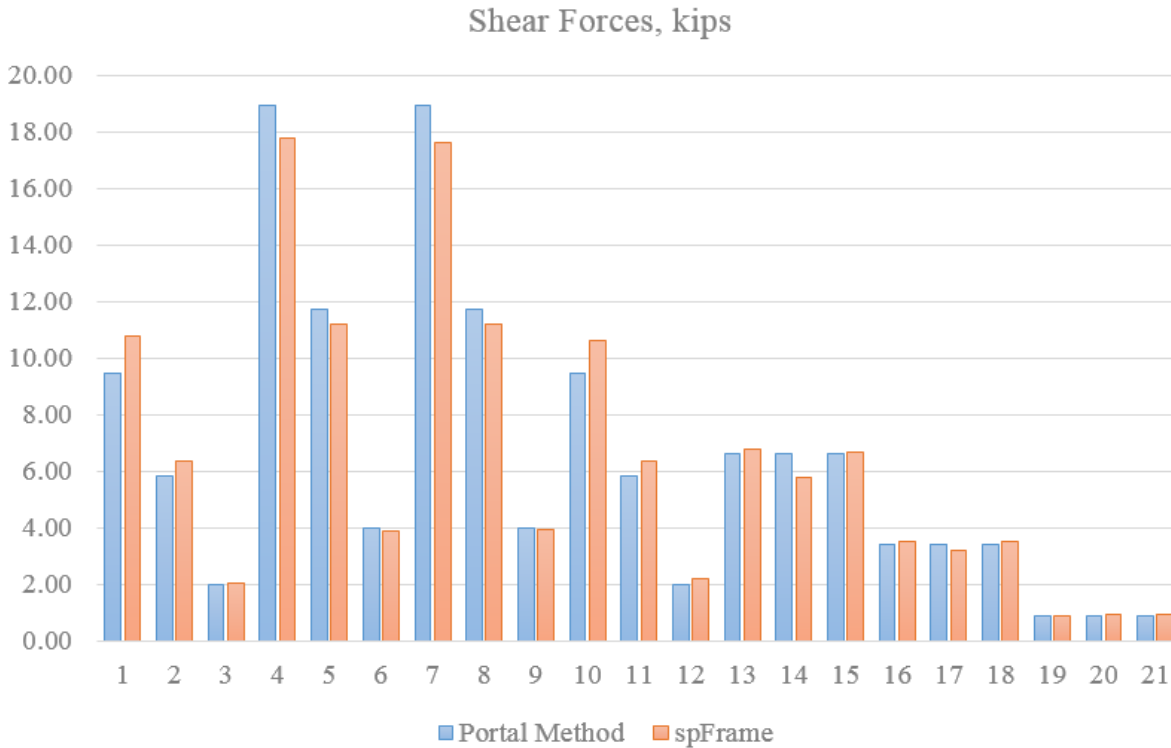


Figure 19 – Shear Forces for Frame #1 ([spFrame](#))

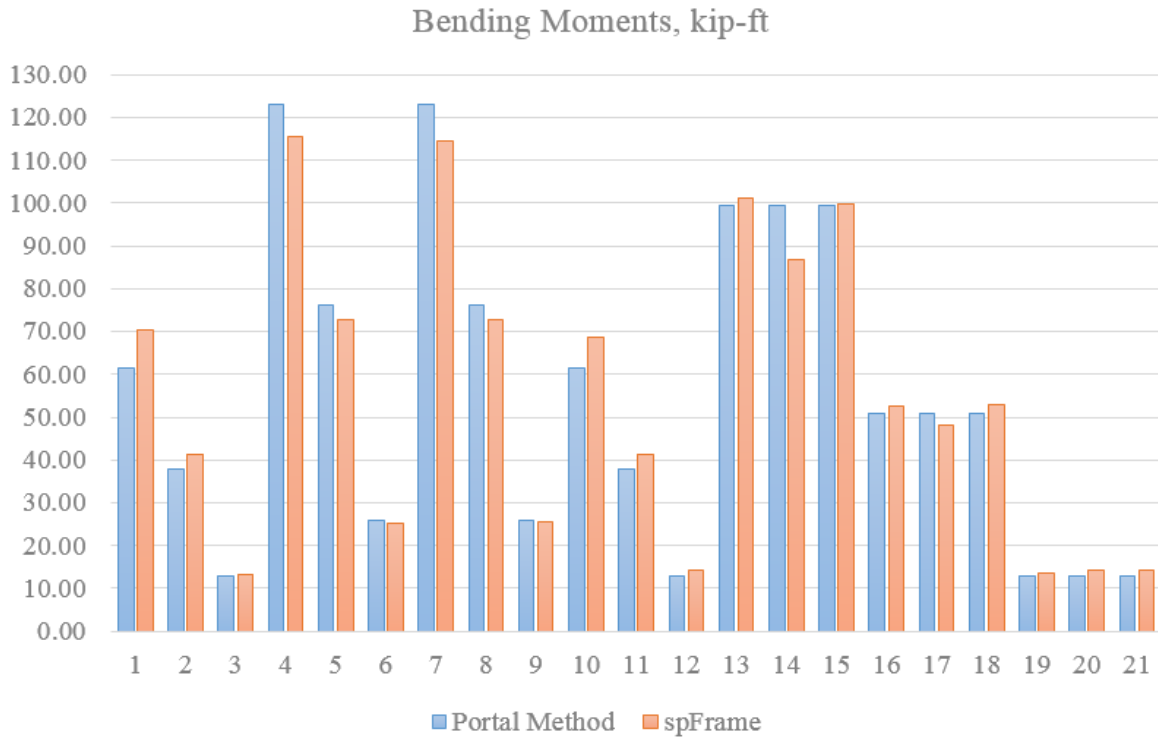


Figure 20 – Bending Moments for Frame #1 (spFrame)

For Frame #2:

|   |    |    |    |    |    |    |
|---|----|----|----|----|----|----|
| 5 | 33 | 10 | 34 | 15 | 34 | 20 |
| 4 | 30 | 9  | 31 | 14 | 31 | 19 |
| 3 | 27 | 8  | 26 | 13 | 28 | 18 |
| 2 | 24 | 7  | 25 | 12 | 25 | 17 |
| 1 | 21 | 6  | 22 | 11 | 23 | 16 |

Figure 21 – Element Numbering for Frame #2 (spFrame)

Table 2 - Comparison of Axial, Shear, and Moment Forces (Frame #2)

| Member | P, kip        |         | V, kip        |         | M, kip-ft     |                |                |         |
|--------|---------------|---------|---------------|---------|---------------|----------------|----------------|---------|
|        | Portal Method | spFrame | Portal Method | spFrame | Portal Method | spFrame        |                |         |
|        |               |         |               |         |               | M <sub>1</sub> | M <sub>2</sub> | Average |
| 1      | 13.38         | 11.70   | 4.58          | 3.68    | 34.38         | 37.20          | 18.00          | 27.60   |
| 2      | 7.80          | 7.85    | 3.57          | 3.45    | 21.40         | 19.60          | 21.80          | 20.70   |
| 3      | 4.10          | 4.45    | 2.60          | 2.48    | 15.60         | 11.80          | 18.00          | 14.90   |
| 4      | 1.59          | 2.02    | 1.58          | 1.54    | 9.50          | 5.96           | 12.50          | 9.23    |
| 5      | 0.32          | 0.63    | 0.53          | 0.62    | 3.20          | 1.23           | 6.16           | 3.70    |
| 6      | 0.00          | 0.70    | 9.17          | 10.10   | 68.75         | 110.00         | 41.80          | 75.90   |
| 7      | 0.00          | 0.47    | 9.90          | 7.23    | 59.38         | 38.10          | 48.70          | 43.40   |
| 8      | 0.00          | 0.30    | 5.20          | 5.32    | 31.20         | 21.50          | 42.40          | 31.95   |
| 9      | 0.00          | 0.17    | 3.17          | 3.22    | 19.00         | 7.92           | 30.70          | 19.31   |
| 10     | 0.00          | 0.08    | 1.07          | 0.97    | 6.40          | 2.01           | 13.70          | 7.86    |
| 11     | 0.00          | 0.70    | 9.17          | 10.10   | 68.75         | 109.00         | 41.60          | 75.30   |
| 12     | 0.00          | 0.46    | 9.90          | 7.25    | 59.38         | 38.20          | 48.70          | 43.45   |
| 13     | 0.00          | 0.30    | 5.20          | 5.32    | 31.20         | 21.50          | 42.40          | 31.95   |
| 14     | 0.00          | 0.16    | 3.17          | 3.21    | 19.00         | 7.91           | 30.70          | 19.31   |
| 15     | 0.00          | 0.08    | 1.07          | 0.98    | 6.40          | 1.92           | 13.70          | 7.81    |
| 16     | 13.38         | 11.70   | 4.58          | 3.66    | 34.38         | 37.00          | 17.90          | 27.45   |
| 17     | 7.80          | 7.86    | 3.57          | 3.47    | 21.40         | 19.70          | 21.90          | 20.80   |
| 18     | 4.10          | 4.45    | 2.60          | 2.48    | 15.60         | 11.70          | 18.00          | 14.85   |
| 19     | 1.59          | 2.03    | 1.58          | 1.53    | 9.50          | 5.95           | 12.40          | 9.18    |
| 20     | 0.32          | 0.64    | 0.53          | 0.63    | 3.20          | 1.32           | 6.23           | 3.78    |
| 21     |               |         | 5.58          | 3.84    | 55.78         | 37.70          | 39.10          | 38.40   |
| 22     |               |         | 5.58          | 4.07    | 55.78         | 40.80          | 40.70          | 40.75   |
| 23     |               |         | 5.58          | 3.83    | 55.78         | 39.10          | 37.60          | 38.35   |
| 24     |               |         | 3.70          | 3.41    | 37.00         | 33.60          | 34.50          | 34.05   |
| 25     |               |         | 3.70          | 3.57    | 37.00         | 35.70          | 35.70          | 35.70   |
| 26     |               |         | 3.70          | 3.41    | 37.00         | 34.50          | 33.60          | 34.05   |
| 27     |               |         | 2.51          | 2.43    | 25.10         | 23.90          | 24.60          | 24.25   |
| 28     |               |         | 2.51          | 2.56    | 25.10         | 25.60          | 25.60          | 25.60   |
| 29     |               |         | 2.51          | 2.43    | 25.10         | 24.60          | 23.90          | 24.25   |
| 30     |               |         | 1.27          | 1.39    | 12.70         | 13.70          | 14.00          | 13.85   |
| 31     |               |         | 1.27          | 1.47    | 12.70         | 14.70          | 14.70          | 14.70   |
| 32     |               |         | 1.27          | 1.39    | 12.70         | 14.00          | 13.70          | 13.85   |
| 33     |               |         | 0.32          | 0.63    | 3.20          | 6.16           | 6.49           | 6.33    |
| 34     |               |         | 0.32          | 0.72    | 3.20          | 7.17           | 7.18           | 7.18    |
| 35     |               |         | 0.32          | 0.64    | 3.20          | 6.54           | 6.23           | 6.39    |

Axial Forces, kips

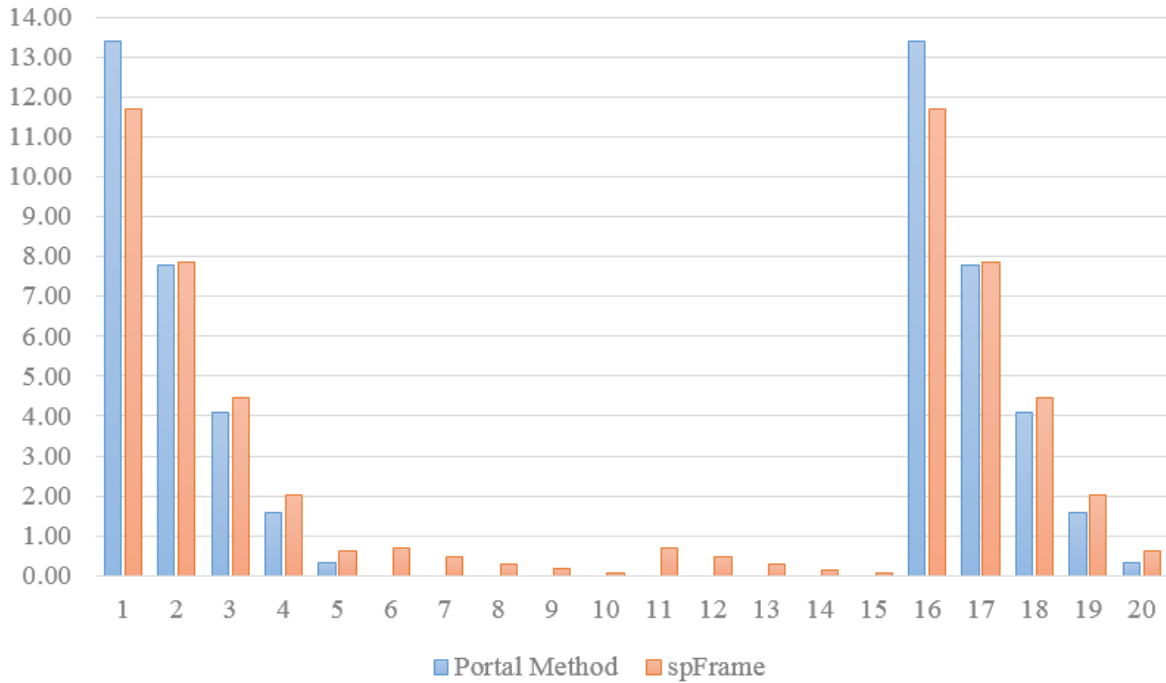


Figure 22 – Axial Forces for Frame #2 (spFrame)

Shear Forces, kips

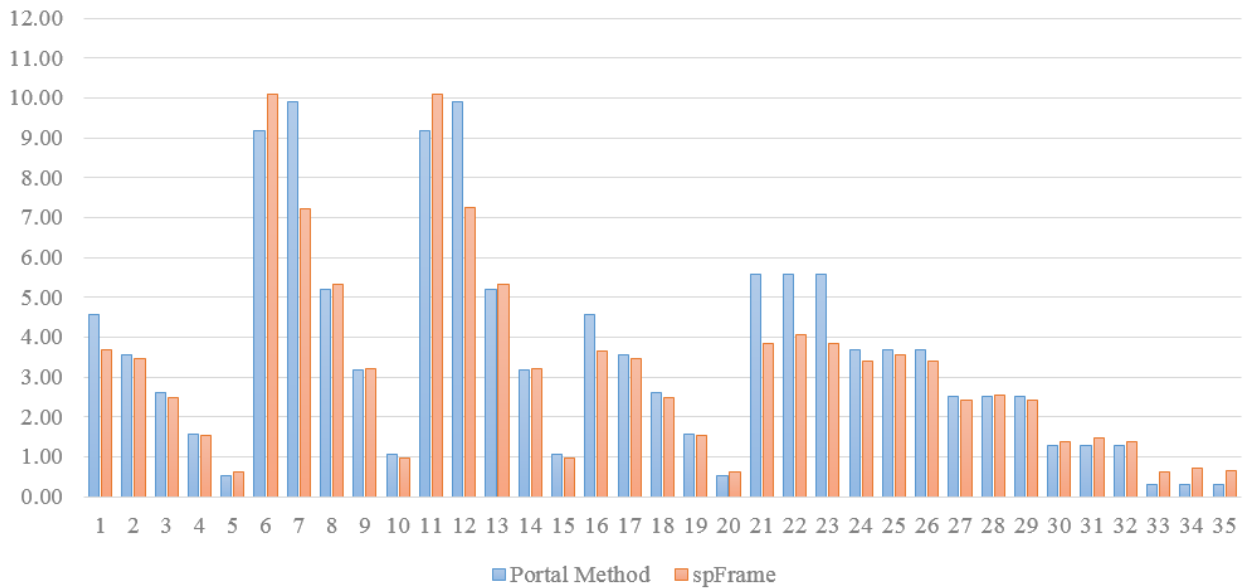


Figure 23 – Shear Forces for Frame #2 (spFrame)

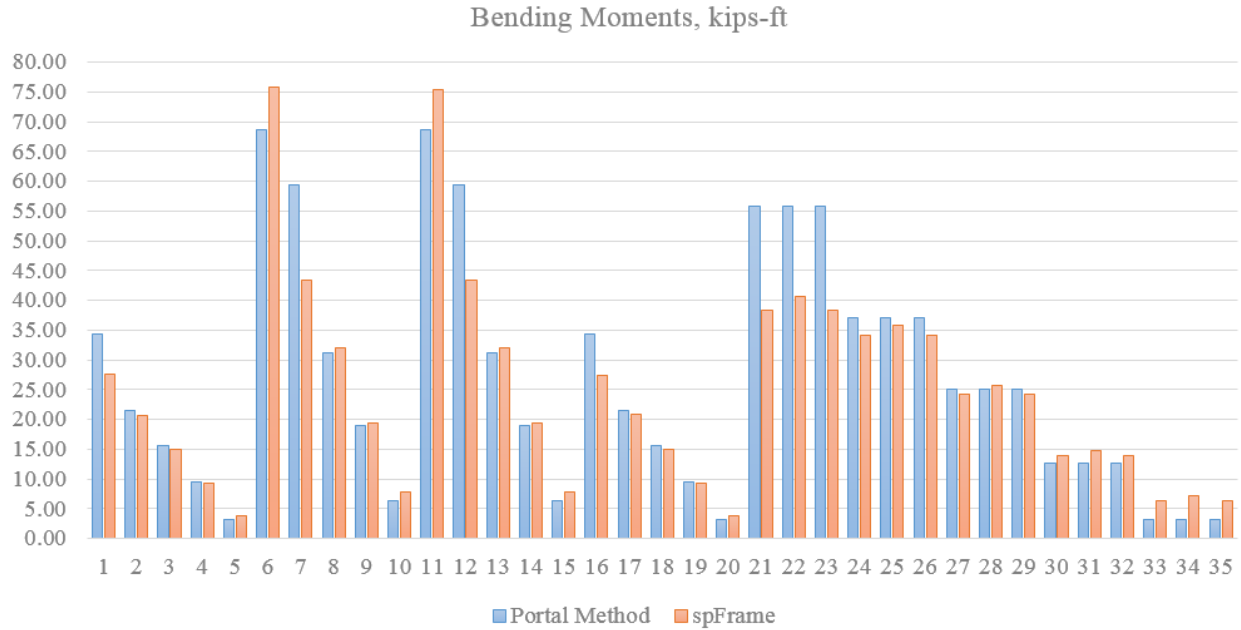


Figure 24 – Bending Moments for Frame #2 (spFrame)

For Frame #3:

|    |    |    |    |    |
|----|----|----|----|----|
| 51 | 52 | 53 | 54 | 55 |
| 5  | 10 | 15 | 20 | 25 |
| 46 | 47 | 48 | 49 | 50 |
| 4  | 9  | 14 | 19 | 24 |
| 41 | 42 | 43 | 44 | 45 |
| 3  | 8  | 13 | 18 | 23 |
| 36 | 37 | 38 | 39 | 40 |
| 2  | 7  | 12 | 17 | 22 |
| 31 | 32 | 33 | 34 | 35 |
| 1  | 6  | 11 | 16 | 21 |
|    |    |    |    | 26 |
|    |    |    |    | 27 |
|    |    |    |    | 28 |
|    |    |    |    | 29 |
|    |    |    |    | 30 |

Figure 25 – Element Numbering for Frame #3 (spFrame)

Table 3 - Comparison of Axial, Shear, and Moment Forces (Frame #3)

| Member | P, kip        |          | V, kip        |         | M, kip-ft     |                |                |         |
|--------|---------------|----------|---------------|---------|---------------|----------------|----------------|---------|
|        | Portal Method | spFrame  | Portal Method | spFrame | Portal Method | spFrame        |                |         |
|        |               |          |               |         |               | M <sub>1</sub> | M <sub>2</sub> | Average |
| 1      | 4.67          | 3.96     | 1.90          | 1.48    | 14.25         | 15.6           | 6.6            | 11.10   |
| 2      | 2.74          | 2.72     | 1.49          | 1.41    | 8.94          | 8.08           | 8.8            | 8.44    |
| 3      | 1.45          | 1.57     | 1.09          | 1.03    | 6.54          | 4.77           | 7.59           | 6.18    |
| 4      | 0.57          | 0.736    | 0.67          | 0.649   | 4.02          | 2.34           | 5.45           | 3.90    |
| 5      | 0.12          | 0.242    | 0.23          | 0.27    | 1.38          | 0.416          | 2.83           | 1.62    |
| 6      | 0.00          | 0.188    | 3.80          | 4.05    | 28.50         | 46             | 14.7           | 30.35   |
| 7      | 0.00          | 0.128    | 2.98          | 2.98    | 17.88         | 16.1           | 19.6           | 17.85   |
| 8      | 0.00          | 0.0838   | 2.18          | 2.2     | 13.08         | 8.51           | 17.9           | 13.21   |
| 9      | 0.00          | 0.048    | 1.34          | 1.35    | 8.04          | 2.75           | 13.5           | 8.13    |
| 10     | 0.00          | 0.0252   | 0.46          | 0.407   | 2.76          | 1.29           | 6.18           | 3.74    |
| 11     | 0.00          | 0.00341  | 3.80          | 4.03    | 28.50         | 45.8           | 14.7           | 30.25   |
| 12     | 0.00          | 0.000633 | 2.98          | 3.04    | 17.88         | 16.5           | 19.9           | 18.20   |
| 13     | 0.00          | 0.000731 | 2.18          | 2.22    | 13.08         | 8.61           | 18.1           | 13.36   |
| 14     | 0.00          | 0.00207  | 1.34          | 1.36    | 8.04          | 2.86           | 13.5           | 8.18    |
| 15     | 0.00          | 0.00104  | 0.46          | 0.445   | 2.76          | 1.08           | 6.42           | 3.75    |
| 16     | 0.00          | 0.00479  | 3.80          | 4.01    | 28.50         | 45.6           | 14.5           | 30.05   |
| 17     | 0.00          | 0.00441  | 2.98          | 3.05    | 17.88         | 16.6           | 19.9           | 18.25   |
| 18     | 0.00          | 0.00371  | 2.18          | 2.22    | 13.08         | 8.59           | 18.1           | 13.35   |
| 19     | 0.00          | 0.00318  | 1.34          | 1.36    | 8.04          | 2.85           | 13.4           | 8.13    |
| 20     | 0.00          | 0.0025   | 0.46          | 0.454   | 2.76          | 1.01           | 6.46           | 3.74    |
| 21     | 0.00          | 0.186    | 3.80          | 3.98    | 28.50         | 45.5           | 14.3           | 29.90   |
| 22     | 0.00          | 0.123    | 2.98          | 3.01    | 17.88         | 16.4           | 19.7           | 18.05   |
| 23     | 0.00          | 0.0799   | 2.18          | 2.2     | 13.08         | 8.45           | 17.9           | 13.18   |
| 24     | 0.00          | 0.0433   | 1.34          | 1.34    | 8.04          | 2.71           | 13.3           | 8.01    |
| 25     | 0.00          | 0.022    | 0.46          | 0.435   | 2.76          | 1.08           | 6.3            | 3.69    |
| 26     | 4.67          | 3.96     | 1.90          | 1.45    | 14.25         | 15.3           | 6.37           | 10.84   |
| 27     | 2.74          | 2.73     | 1.49          | 1.42    | 8.94          | 8.22           | 8.85           | 8.54    |
| 28     | 1.45          | 1.58     | 1.09          | 1.03    | 6.54          | 4.74           | 7.59           | 6.17    |
| 29     | 0.57          | 0.746    | 0.67          | 0.641   | 4.02          | 2.32           | 5.37           | 3.85    |
| 30     | 0.12          | 0.249    | 0.23          | 0.288   | 1.38          | 0.539          | 2.92           | 1.73    |
| 31     |               |          | 1.93          | 1.24    | 23.19         | 14.7           | 15.2           | 14.95   |
| 32     |               |          | 1.93          | 1.3     | 23.19         | 15.6           | 15.6           | 15.60   |
| 33     |               |          | 1.93          | 1.3     | 23.19         | 15.6           | 15.6           | 15.60   |
| 34     |               |          | 1.93          | 1.3     | 23.19         | 15.6           | 15.6           | 15.60   |
| 35     |               |          | 1.93          | 1.24    | 23.19         | 15.1           | 14.6           | 14.85   |
| 36     |               |          | 1.29          | 1.14    | 15.48         | 13.6           | 13.9           | 13.75   |
| 37     |               |          | 1.29          | 1.19    | 15.48         | 14.3           | 14.3           | 14.30   |



| Member | P, kip        |         | V, kip        |         | M, kip-ft     |                |                |         |
|--------|---------------|---------|---------------|---------|---------------|----------------|----------------|---------|
|        | Portal Method | spFrame | Portal Method | spFrame | Portal Method | spFrame        |                |         |
|        |               |         |               |         |               | M <sub>1</sub> | M <sub>2</sub> | Average |
| 38     |               |         | 1.29          | 1.19    | 15.48         | 14.3           | 14.3           | 14.30   |
| 39     |               |         | 1.29          | 1.19    | 15.48         | 14.3           | 14.3           | 14.30   |
| 40     |               |         | 1.29          | 1.15    | 15.48         | 13.9           | 13.6           | 13.75   |
| 41     |               |         | 2.51          | 0.837   | 25.10         | 9.93           | 10.2           | 10.07   |
| 42     |               |         | 2.51          | 0.873   | 25.10         | 10.5           | 10.5           | 10.50   |
| 43     |               |         | 2.51          | 0.872   | 25.10         | 10.5           | 10.5           | 10.50   |
| 44     |               |         | 2.51          | 0.872   | 25.10         | 10.5           | 10.5           | 10.50   |
| 45     |               |         | 2.51          | 0.836   | 25.10         | 10.2           | 9.91           | 10.06   |
| 46     |               |         | 0.45          | 0.494   | 5.40          | 5.86           | 5.99           | 5.93    |
| 47     |               |         | 0.45          | 0.517   | 5.40          | 6.2            | 6.2            | 6.20    |
| 48     |               |         | 0.45          | 0.518   | 5.40          | 6.21           | 6.22           | 6.22    |
| 49     |               |         | 0.45          | 0.518   | 5.40          | 6.22           | 6.22           | 6.22    |
| 50     |               |         | 0.45          | 0.497   | 5.40          | 6.03           | 5.91           | 5.97    |
| 51     |               |         | 0.12          | 0.242   | 1.38          | 2.83           | 2.97           | 2.90    |
| 52     |               |         | 0.12          | 0.267   | 1.38          | 3.2            | 3.2            | 3.20    |
| 53     |               |         | 0.12          | 0.268   | 1.38          | 3.21           | 3.22           | 3.22    |
| 54     |               |         | 0.12          | 0.271   | 1.38          | 3.24           | 3.25           | 3.25    |
| 55     |               |         | 0.12          | 0.249   | 1.38          | 3.05           | 2.92           | 2.99    |

Axial Forces, kips

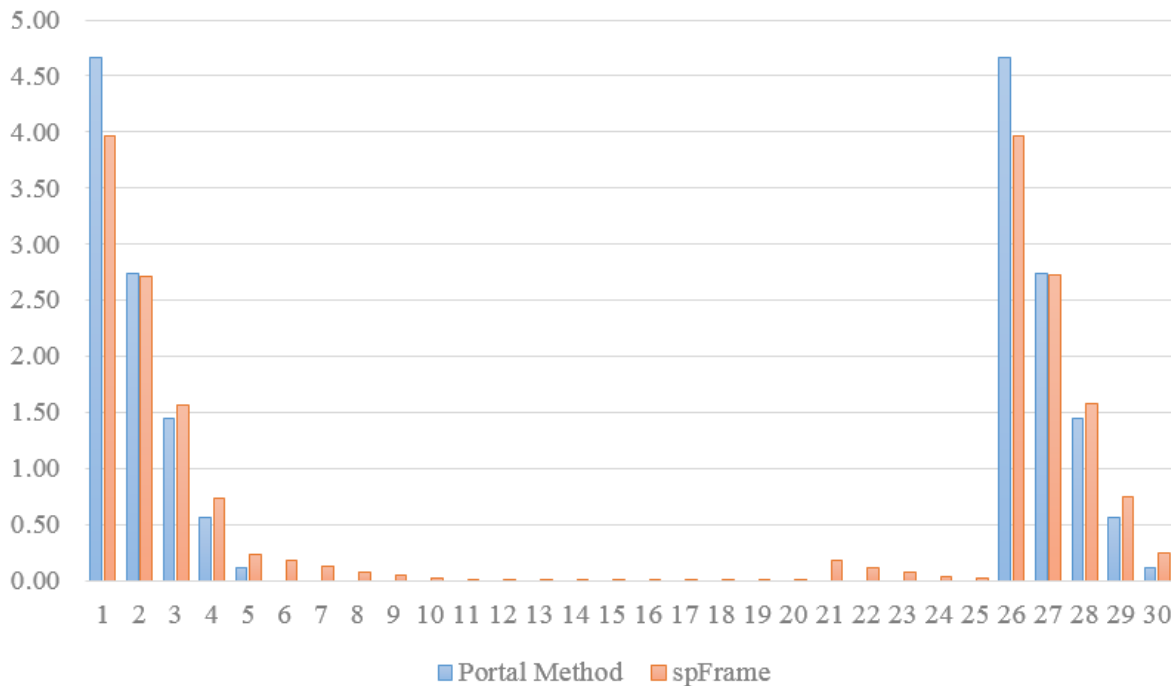


Figure 26 – Axial Forces for Frame #3 (spFrame)

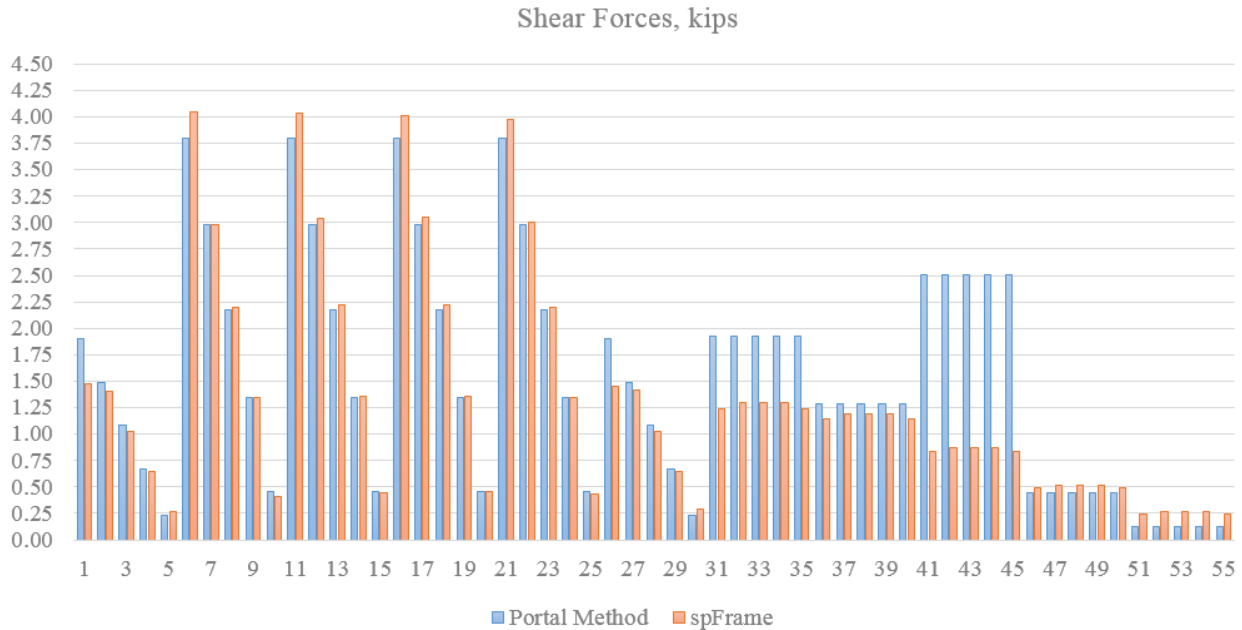


Figure 27 – Shear Forces for Frame #3 ([spFrame](#))

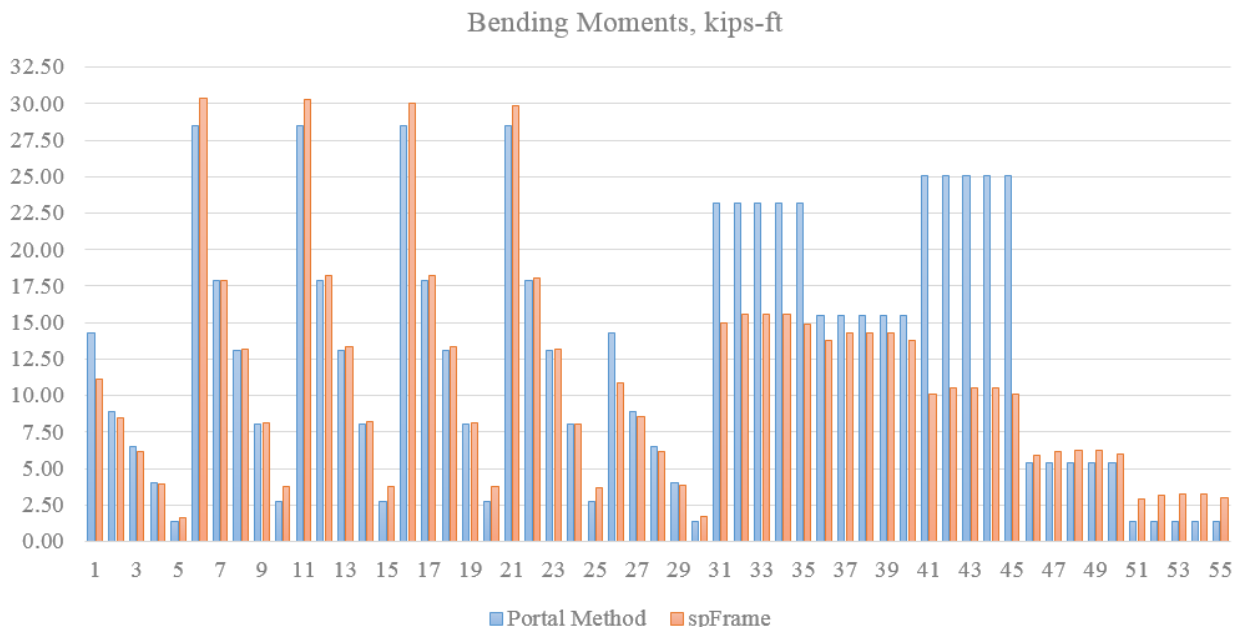


Figure 28 – Bending Moments for Frame #3 ([spFrame](#))

The results of all the hand calculations and the reference used illustrated above are in agreement with the automated exact results obtained from the [spFrame](#) program. It is noticed from the results that the simplified method to analyze the frame is accurate. However, the level of accuracy decreases with the increase of the frame complexity as shown in the previous Figures. The use of simplified methods for frame analysis should be accompanied with caution and engineering judgement. The use of advanced analysis techniques (such as the stiffness method used in [spFrame](#)) is a powerful tool to perform frame analysis for any number of joints and elements with accurate results.